Kiselev's
GEOMETRY

Book I. PLANIMETRY

by

A. P. Kiselev

Adapted from Russian
by Alexander Givental

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Department of Mathematics, The University of Rochester.

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Translator's Foreword

Those reading these lines are hereby summoned to raise their children to a good command of Elementary Geometry, to be judged by the rigorous standards of the ancient Greek mathematicians.

A magic spell

Mathematics is an ancient culture. It is passed on by each generation to the next. What we now call *Elementary Geometry* was created by Greeks some 2300 years ago and nurtured by them with pride for about a millennium. Then, for another millennium, Arabs were preserving Geometry and transcribing it to the language of *Algebra* that they invented. The effort bore fruit in the Modern Age, when exact sciences emerged through the work of Frenchman Rene Descartes, Englishman Isaac Newton, German Carl Friedrich Gauss, and their contemporaries and followers.

Here is one reason. On the decline of the 19th century, a Scottish professor showed to his class that the mathematical equations, he introduced to explain electricity experiments, admit wave-like solutions. Afterwards a German engineer Heinrich Hertz, who happened to be a student in that class, managed to generate and register the waves. A century later we find that almost every thing we use: GPS, TV, cell-phones, computers, and everything we manufacture, buy, or learn using them, descends from the mathematical discovery made by James Clerk Maxwell.

I gave the above speech at a graduation ceremony at the University of California Berkeley, addressing the class of graduating math majors — and then I cast a *spell* upon them.

Soon there came the realization that without a Magic Wand the spell won't work: I did not manage to find any textbook in English that I could recommend to a young person willing to master Elementary Geometry. This is when the thought of Kiselev's came to mind.

Andrei Petrovich Kiselev (pronounced And-′rei Pet-′ro-vich Ki-se-′lyov) left a unique legacy to mathematics education. Born in 1852 in a provincial Russian town Mzensk, he graduated in 1875 from the Department of Mathematics and Physics of St.-Petersburg University to begin a long career as a math and science teacher and author. His school-level textbooks "A Systematic Course of Arithmetic" [1] [9], "Elementary Algebra" [10], and "Elementary Geometry"(Book I "Planimetry", Book II "Stereometry") [3] were first published in 1884, 1888 and 1892 respectively, and soon gained a leading position in the Russian mathematics education. Revised and published more than a hundred times altogether, the books retained their leadership over many decades both in Tsarist Russia, and after the Revolution of 1917, under the quite different cultural circumstances of the Soviet epoch. A few years prior to Kiselev's death in 1940, his books were officially given the status of *stable*, i.e. main and only textbooks to be used in all schools to teach all teenagers in the totalitarian state with a 200-million population. The books held this status until 1955 (and "Stereometry" even until 1974) when they got replaced in this capacity by less successful clones written by more Soviet authors. Yet "Planimetry" remained the favorite under-the-desk choice of many teachers and a must for honors geometry students. In the last decade, Kiselev's "Geometry," which has long become a rarity, was reprinted by several major publishing houses in Moscow and St.-Petersburg in both versions: for teachers [6, 8] as an authentic pedagogical heritage, and for students [5, 7] as a textbook tailored to fit the currently active school curricula. In the post-Soviet educational market, Kiselev's "Geometry" continues to compete successfully with its own grandchildren.

What is the secret of such ageless vigor? There are several.

Kiselev himself formulated the following three key virtues of good textbooks: *precision*, *simplicity*, *conciseness*. And *competence in the subject* — for we must now add this fourth criterion, which could have been taken for granted a century ago.

Acquaintance with programs and principles of math education being developed by European mathematicians was another of Kiselev's assets. In his preface to the first edition of "Elementary Geometry," in addition to domestic and translated textbooks, Kiselev quotes ten geometry courses in French and German published in the previous decade.

Yet another vital elixir that prolongs the life of Kiselev's work was the continuous effort of the author himself and of the editors of later reprints to improve and update the books, and to accommodate the teachers' requests, curriculum fluctuations and pressures of the 20th century classroom.

Last but not least, deep and beautiful geometry is the most efficient preservative. Compared to the first textbook in this subject: the "Elements" [1], which was written by *Euclid of Alexandria* in the 3rd century B.C., and whose spirit and structure are so faithfully represented in Kiselev's "Geometry," the latter is quite young.

Elementary geometry occupies a singular place in secondary education. The acquiring of superb reasoning skills is one of those benefits from study-

[1] The numbers in brackets refer to the bibliography on p. 235.

ing geometry whose role reaches far beyond mathematics education *per se*. Another one is the unlimited opportunity for nurturing creative thinking (thanks to the astonishingly broad difficulty range of elementary geometry problems that have been accumulated over the decades). Fine learning habits of those who dared to face the challenge remain always at work for them. A lack thereof in those who missed it becomes hard to compensate by studying anything else. Above all, elementary geometry conveys the essence and power of the *theoretical method* in its purest, yet intuitively transparent and aesthetically appealing, form. Such high expectations seem to depend however on the appropriate framework: a textbook, a teacher, a culture.

In Russia, the adequate framework emerged apparently in the mid-thirties, with Kiselev's books as the key component. After the 2nd World War, countries of Eastern Europe and the Peoples Republic of China, adapted to their classrooms math textbooks based on Soviet programs. Thus, one way or another, Kiselev's "Geometry" has served several generations of students and teachers in a substantial portion of the planet. It is the time to make the book available to the English reader.

"Planimetry," targeting the age group of current 7–9th-graders, provides a concise yet crystal-clear presentation of elementary plane geometry, in all its aspects which usually appear in modern high-school geometry programs. The reader's mathematical maturity is gently advanced by commentaries on the nature of mathematical reasoning distributed wisely throughout the book. Student's competence is reinforced by generously supplied exercises of varying degree of challenge. Among them, *straightedge and compass* constructions play a prominent role, because, according to the author, they are essential for animating the subject and cultivating students' taste. The book is marked with the general sense of measure (in both selections and omissions), and non-cryptic, unambiguous language. This makes it equally suitable for independent study, teachers' professional development, or a regular school classroom. The book was indeed designed and tuned to be *stable*.

Hopefully the present adaptation retains the virtues of the original. I tried to follow it pretty closely, alternating between several available versions [3, 4, 5, 7, 8] when they disagreed. Yet authenticity of translation was not the goal, and I felt free to deviate from the source when the need occurred.

The most notable change is the significant extension and rearrangement of exercise sections to comply with the US tradition of making textbook editions self-contained (in Russia separate problem books are in fashion).

Also, I added or redesigned a few sections to represent material which found its way to geometry curricula rather recently.

Finally, having removed descriptions of several obsolete drafting devices (such as a pantograph), I would like to share with the reader the following observation.

In that remote, Kiselevian past, when Elementary Geometry was the most reliable ally of every engineer, the straightedge and compass were the main items in his or her drafting toolbox. The craft of blueprint drafting has long gone thanks to the advance of computers. Consequently, all 267 diagrams in the present edition are produced with the aid of graphing software *Xfig*. Still, Elementary Geometry is manifested in their design in multiple ways. Obviously, it is inherent in all modern technologies through the "custody chain": Euclid – Descartes – Newton – Maxwell. Plausibly, it awakened the innovative powers of the many scientists and engineers who invented and created computers. Possibly, it was among the skills of the authors of *Xfig*. Yet, symbolically enough, the most reliable way of drawing a diagram on the computer screen is to use electronic surrogates of the straightedge and compass and follow literally the prescriptions given in the present book, often in the very same theorem that the diagram illustrates. This brings us back to Euclid of Alexandria, who was the first to describe the theorem, and to the task of passing on *his* culture.

I believe that the book you are holding in your hands gives everyone a fair chance to share in the "custody." This is my Magic Wand, and now I can cast my spell.

Alexander Givental
Department of Mathematics
University of California Berkeley
April, 2006

Authors cited in this book:

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Eudoxus of Cnidus	***408 – 355 B.C.***
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Archimedes of Syracuse	***287 – 212 B.C.***
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