Chapter 1

LINES AND PLANES

1 Drawing a plane

1. Preliminary remarks. In **stereometry** (called also **solid geometry**) one studies geometric figures not all of whose elements fit the same plane.

Geometric figures in space are shown on the plane of a diagram following certain conventions, intended to make the figures and their diagrams appear alike.

Many real objects around us have surfaces which resemble geometric planes and are shaped like rectangles: the cover of a book, a window pane, the surface of a desk, etc. When seen at an angle and from a distance, such surfaces appear to have the shape of a parallelogram. It is customary, therefore, to show a plane in a diagram as a parallelogram. The plane is usually denoted by one letter, e.g. "the plane M" (Figure 1).

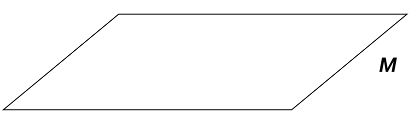

Figure 1

2. Basic properties of the plane. Let us point out the following properties of planes, which are accepted without proof, i.e. considered axioms.

(1) *If two points*[1] *of a line lie in a given plane, then every point of the line lies in this plane.*

(2) *If two planes have a common point, then they intersect in a line passing through this point.*

(3) *Through every three points not lying on the same line, one can draw a plane, and such a plane is unique.*

3. `Corollaries.` (1) *Through a line and a point outside it, one can draw a plane, and such a plane is unique.* Indeed, the point together with any two points on the line form three points not lying on the same line, through which a plane can therefore be drawn, and such a plane is unique.

(2) *Through two intersecting lines, one can draw a plane, and such a plane is unique.* Indeed, taking the intersection point and one more point on each of the lines, we obtain three points through which a plane can be drawn, and such a plane is unique.

(3) *Through two parallel lines, one can draw only one plane.* Indeed, parallel lines, by definition, lie in the same plane. Such a plane is unique, since through one of the lines and any point of the other line, at most one plane can be drawn.

4. Rotating a plane about a line. *Through each line in space, infinitely many planes can be drawn.*

Indeed, let a line a be given (Figure 2). Take any point A outside it. Through the line a and the point A, a unique plane is passing (§3). Let us call this plane M. Take a new point B outside the plane M. Through the line a and the point B, too, a unique plane is passing. Let us call this plane N. It cannot coincide with M, since it contains the point B which does not lie in the plane M. Furthermore, we can take in space yet another point C outside the planes M and N. Through the line a and the point C, yet a new plane is passing. Denote it P. It coincides neither with M nor with N, since it contains the point C which lies neither in the plane M nor in the plane N. Proceeding by taking more and more points in space, we will thus obtain more and more planes passing through the given line a. There will be *infinitely many* such planes. All these planes can be considered as various positions of the same plane which *rotates* about the line a.

We may therefore formulate one more property of the plane: *a plane can be rotated about every line lying in this plane.*

[1] As in Book I, we will always assume that expressions like "three points," "two planes," etc. refer to *distinct* points, planes, etc.